WHO WAS JAMES CLERK MAXWELL AND WHAT WAS/IS HIS ELECTROMAGNETIC THEORY

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ABSTRACT - The Electromagnetic community make profuse utilization of Maxwell's equations, his theory and their applications. It is arguable that very few of us have clear ideas about what exactly Maxwell did and what kind of scientist he was.

In fact, he developed many of the fundamental ideas in electrical engineering and provided mathematical language for their exposition. His contributions to other branches of science are no less significant. He was not only one of the great scientists of the nineteenth century but also is great for all time. To this end, the present essay starts with a brief outline of his life; and then it provides a short but critical discussion of his original contributions in electromagnetics and their evolution as his electromagnetic theory. We also give a cursory review of his significant contributions in other areas of science. It is hoped that this will provide the electromagnetic community readers with better and more complete appreciation of James Clerk Maxwell as a scientist as well as of his electromagnetic theory as we know now it.

Key Words: Maxwell, Hertz, Heaviside, Faraday, Ampère, Coulomb, Fitzgerald, Lodge, Larmor, Cavendish, Inverse squared law, Information theory, Control theory, Color vision, Electromagntic theory, probability theory, entropy, chromatist diagram, television, aether, light, system of units, kinetic theory of gases, normal distribution, velocity of light, index of refraction, quaternion, dimensional analysis, radiation pressure, Rings of Saturn, Kirchoff's circuital laws, Curl, Divergence, Gradient, Maxwell's demon, lightning protection, color photograph, stereoscope, Cybernetics.

1. Introduction: It is arguable that very few of us in engineering and, particularly in the electromagnetic community, whose professional life blood is sustained by Maxwell's equations and related ideas possess clear conception as to what exactly Maxwell did and, in fact, what was his electromagnetic theory. It is not improbable that many of us are not aware of his recognized significant contributions in other areas of science.

In this paper we discuss briefly how Maxwell arrived at some of his famous conclusions and speculations from which his electromagnetic theory evolved. In addition, we also give a short overview of his significant contributions in physics and mathematics for which he is also rightfully recognized. For obvious reasons, here we emphasize Maxwell's work in electromagnetics. We hope our paper will help the reader to appreciate Maxwell better, as one of the prominent physicists of all times.

2. A Biographical Sketch of James Clerk Maxwell:

John Clerk Maxwell, his father, had added the name Maxwell to that of Clerk on inheriting an estate. His mother was Frances Cay. Maxwell was born in Edinburgh at 14 India Street on June 13, 1831. He spent his infancy and early boyhood at Glenlair, a house built by his father shortly before his marriage. He was always inquisitive and throughout his childhood his constant question was "What's the go of that?", "What does it do?" and when he was not satisfied with the answer he would ask "But what is the particular go of it?" Besides asking questions he was very fond of making things such as baskets and seals covered with strange devices. His mother died when he was nine years old, of the same disease which killed him forty years later. At age 10, he went to the Edinburgh academy. He used to go to school wearing clothes designed by his father and his friends made fun of him. He was shy and dull according to Professor P. G. Tait, one year his junior. He made no friends. However, in the middle of his school career he suddenly became one of the most brilliant students. At the age of 14, he published his first paper in Proceedings of the Royal Society of Edinburgh. During his school days he used to play the game Diabolo, which he played throughout his life. Diabolo is translated into devil on two sticks. The devil consists of a double cone, the narrow part resting on a string whose ends are attached to the end of two sticks which are held in the hands of the player and by moving the ends of the sticks in opposite direction it is possible to give a considerable rotation to the devil – a home made gyroscope. Maxwell spent six years at the academy and then went for 3 years to the University of Edinburgh. On leaving Edinburgh in 1850, he went to Cambridge entering first at Peterhouse and then after one term moved to Trinity as the chance for a mathematician for obtaining a fellowship was greater in the latter. On his second year when he obtained the scholarship, he also was made one of the Apostles – a club limited to 12 members, and according to themselves they were the men with outstanding ability in the University. Dr. Butler, a master at Trinity said: The impression of power which Maxwell produced on all he met was remarkable; it was often much more due to his personality than to what he said, for many found it difficult to follow him in his quick changes from one subject to the another, his lively imagination started so many hares that before he had run one down he was off after another. In January 1854 he took the mathematical Tripos (the different kinds of honours for the bachelor degree at the University of Cambridge) and was the second wrangler behind Routh. After his degree Maxwell stayed at Trinity and obtained a Fellowship in 1855. He became the Professor of Natural philosophy (i.e., physics) at Marischal College at Aberdeen during the time of his father's death. In February 1858, he married Katherine Mary Dewar, the daughter of the Principal of Marischal College. In 1860, King's College of London and Marischal College merged and the professorship of physics at Marischal College was eliminated and he was appointed a few months later to the professorship of Natural philosophy at King's College, London. His wife used to help him in carrying out his various experiments. After five years there, in 1865, he resigned and went back to Glenlair. In 1871 he returned to Cambridge when a Professorship in Experimental Physics was established due to a sizable grant by the seventh Duke of Devonshire, a grandson of Henry Cavendish. When

Maxwell, assumed the Professorship at Cambridge he came across Cavendish's unpublished pile of manuscripts and he decided to edit them himself as he found in the manuscripts that Cavendish had already found the inverse square law of electrostatic repulsion and even have verified it experimentally around 1771 much before Coulomb in the 1780s. Maxwell was thus the first director of the famous Cavendish laboratory in Cambridge. Cavendish's work was published in 1879, the year Maxwell was diagnosed with stomach cancer and the doctors told him that he had a month to live. He died on November 5, 1879 [8].

3. Significant Contributions of Maxwell: The name of James Clerk Maxwell is synonymous with the development of modern physics. He laid the basic foundation for electricity, magnetism, and optics [1-16]. The theory of electromagnetism is one of the few theories where the equations have not changed since its original conception, although their interpretations have gone through revolutionary changes at least twice. The first revolution was by Heinrich Rudolf Hertz and Oliver Heaviside and the second by Joseph Larmor, through the introduction of the electron into the theory. Maxwell's work on electromagnetic theory was only a small part of his research. As Sir James Hopwood Jeans (1877–1946) pointed out [4, p.91]: *In his hands electricity first became a mathematically exact science and the same might be said of other larger parts of Physics*. In whatever area he worked, he brought new innovation. He published five books and approximately 100 papers. Maxwell can be considered as one of the greatest scientists of the world even if he had never worked on electricity and magnetism. His influence is everywhere, which surprisingly is quite unknown to most engineers!

Here we provide *a cursory overview of some of his achievements*, which are still in vogue today. Hopefully, this will make one more familiar with Maxwell's other scientific achievement, as electromagnetics took shape under him. The name of Maxwell invokes laudatory attributes by famous scientists, for example: *One scientific epoch ended and another began with James Clerk Maxwell* – Albert Einstein, and *From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics* – Richard Phillips Feynman. Henry Augustus Rowland, the first president of the American Physical society after his meeting with Maxwell wrote to Daniel Coit Gilman, President of the Johns Hopkins University, reporting on his encounters with British physicists: *As for professors I have yet met, they are men like the rest of us! After seeing Maxwell I felt somewhat discouraged for here I met with a mind whose superiority was almost oppressive* [3, pp.3]

Maxwell's first publication *On the Description of Oval Curves, and Those Having a Plurality of Foci* was done at the age of 14 in 1846 [1, pp. 35-42, 5, pp. 1-3]. It dealt with drawing curves on a piece of paper using pins, string, and pencil. He varied the number of times he looped the string around each pin to generate various egg shaped graphs. An example of this construct is shown in Figure 1. It turned out that such constructions have significant practical applications in optics. In 1855, his paper on *Description of a New Form of Platometer, an Instrument for Measuring the Area of Plane Figures Drawn on a Paper* [1, pp.275-279, 5, pp. 230-287], provided such an instrument (for measuring areas of plane figures drawn on a paper).

In 1847, his uncle John Cay introduced him to William Nicol who introduced him to the study of polarized light. This led him to study the chromatic effects of polarized light in crystal and strained glass [1, pp. 4]. However, his studies broadened to include the measurement of the coefficients of elasticity of rods and wires and to consider the determination of the compressibility of water. He wrote papers on that topic [1, pp. 120-127] as we shall see later.

In 1855, he published the first-part of his paper *On Faraday's Lines of Force* [1, pp. 337-375, 5, pp. 155-229], where he used fluid flow as an analogy for lines of force shaped in his exposition based on physical geometry. His objective was to find a physical analogy which would help the mind to grasp the results of previous investigations without being committed to any theory founded on the physical science from which that conception is borrowed so that it is neither drawn aside from the subject in the pursuit of analytical subtleties nor carried beyond the truth by a favorite hypothesis. The laws of electricity were compared with the properties of an incompressible fluid, the motion of which was retarded by a force proportional to the velocity, and the fluid was supposed to posses no inertia. The geometrical relations between lines and surfaces were crucial elements in Maxwell's field theory of electricity and magnetism, where the use of vectors, integral theorems and topology became central to his mathematical method [1, pp. 14]. Here, he introduced the concept of image theory for efficient calculation of the fields. Maxwell provided the mathematical basis of the theory of electric images which was earlier developed by Sir William Thomson (better known as Lord Kelvin) to illustrate that equivalent fields can be obtained from them [7, pp. 226-227]. He applied his analogy to the theory of electrostatics, dielectrics, paramagnetism and diamagnetism to Michael Faraday's ideas of magnetic properties of crystalline materials and to electric currents. The second part of his paper proposed a theory based on his distinction between electric and magnetic quantities (acting through surfaces) and intensities acting along lines [1, pp. 15 & 371]. Maxwell, thus proceeded to consider the phenomenon of Electromagnetism and showed how the laws discovered by André Marie Ampère lead to conclusions identical to those of Faraday.

In 1855, he published a paper on *Experiments on Colour as Perceived by the Eye, with Remarks on Colour Blindness* [1, pp. 287, 5, pp. 126-154]. The first consequence of this theory was that between any four colors an equation can be found, and this is confirmed by experiments. Secondly, from two equations containing different colors a third may be obtained. A graphical method can be described by which, after fixing arbitrarily the position of three standard colors, that of any other color can be obtained by experiments. Finally, the effect of red and green glasses on the color-blind was described, and a pair of spectacles having one glass red and the other green was proposed as assistance to them to detect doubtful colors. He was the first to show that in color blind people, their eyes are sensitive only to two colors and not to three as in normal eyes. Typically, they are not sensitive to red. In addition, he showed in his paper *On Colour Vision* [6, pp. 267-279] that color blind people can only experience the blue and green and not the red. He wrote in excess of ten papers on this topic. One of his objectives was to study the sensitivity of the retina to color. To achieve this he developed equipments to look into the eye which were a modification of the ophthalmoscope originally developed by Hermann Ludwig Ferdinand von Helmholtz. At the point of the retina where it is intersected by the axis of the eye there is a *yellow spot*, called the *macula*. Maxwell observed that the nature of the spot changes as a function of the quality of the vision. The macular degeneration of the eye affects the quality of vision and is the leading cause of blindness in people over 55 years old. Today, the extent of macular degeneration of the retina is characterized by *Maxwell's yellow spot test*. He also developed the fish eye lens to look into the retina with little trauma.

Maxwell worked on the generation of white light by mixing different colors and in 1860, published the paper *On the Theory of Compound Colours and its Relations to the Colours of the Spectrum* [1, pp. 633-639, 5, pp. 410-444]. In this paper, he extended the work of Thomas Young who first postulated only three colors, red, green and violet are necessary to produce any color including white and not all the colors of the spectrum are necessary as first illustrated by

Newton. He also incorporated Hermann Gűnther Grassman's concept that there are three variables of color vision (spectral color, intensity of illumination and the degree of saturation). Maxwell showed that these color variables can be represented on a color diagram based on three primary colors [1, pp. 17]. While Newton distinguished his principal colors from the painters triad of primary colors (red, yellow and blue), he supposed the identity of mixing rule for lights and pigments. Even though Helmholtz explained that the mixture of color lights is an additive process while the mixture of pigments is a subtractive process as illustrated in Figure 2, Maxwell made experiments and developed a complete theory to explain how this happens by creating a color triangle which was originally suggested by James David Forbes [1, pp. 16] and illustrated that any color can be generated with a mixture of any three primary colors and that a normal eye has three sorts of receptors as illustrated in his 1861 paper *On the Theory of Three Primary Colours* [5, pp. 445-450]. He chose the three primary colors as red, green, and blue.

Maxwell asked Thomas Sutton in 1861 (the inventor of the single lens reflex camera patented by Sutton also in 1861) to take the first color photograph of a Tartan ribbon shown in Figure 3. The experimental set up was to take three pictures separately using different colors and then project the superposed picture to generate a color photograph. Today, color television works on this principle, but Maxwell's name is rarely mentioned. However, other choices for the primary colors are equally viable. He provided a methodology for generating any color represented by a point inside a triangle whose vertices represented the three primary colors. The new color is generated by mixing the three primary colors in a ratio determined by the respective distance of the point representing the new color from the vertices of the triangle. In the present time, this triangle is called a chromatist diagram as shown in Figure 4 and differs in details from his original. He used polarized light to reveal strain patterns in mechanical structures and developed a graphical method for calculating the forces in the framework. He used a color box to illustrate the generation of any color in the spectrum. Maxwell also demonstrated that for the painters the process is subtractive instead of additive as shown in Figure 2. He demonstrated these principles by pasting different color papers on a spinning top.

On the General Laws of Optical Instruments [5, pp. 271-286, 9, 10] published in 1858, Maxwell laid down the conditions which a perfect optical instrument must fulfill, and showed that if an instrument produced a perfect image of an object, i.e., image free from astigmatism, curvature and distortion, for two different positions of the object, it will give a perfect image at all distances. Maxwell worked on the principles of geometrical optics and based on his work proposed a new theory of optical instruments in which theorems expressing geometrical relations between an object and an image are separated from discussion of the dioptrical properties of lenses [1, pp. 385-398]. This led him to the design of a reflecting stereoscope.

His paper in 1858 *On the Stability of Motion of Saturn's Rings* [1, pp. 438-479, 5, pp. 288-376] won him the Adams prize {named after the English astronomer John Couch Adams (1819–1892), the discoverer of Neptune}. It took him two years to do this work. The problem that he addressed was under what conditions the rings of Saturn would be stable if they were (1) solid, (2) fluid, or (3) composed of many separate pieces of matter. Maxwell showed that the only possible solution is that the ring be consisted of separate bodies. When he considered only two rings, one inside the other, he found that some arrangements were stable, but others not. Also, because of friction, he predicted that the inner ring will move inward and the outer ring outward on a very long time scale. Sir George Biddell Airy said that this was one of the most remarkable applications of mathematics he has ever seen. His conclusions received observational confirmation 38 years later when James Edward Keeler (1857–1900) obtained spectroscopic proof that the outermost portions of the rings were rotating less rapidly than the inner portions. He also built a hand cranked model to demonstrate the motion of the rings. Some say this research led him directly to the kinetic theory of gases.

He worked on the kinetic theory of gases and published a paper in 1860 on *Illustrations of the Dynamical Theory of Gases* [5, pp. 377-409]. He started an interest on gas viscosity, relevant to establishing the effect of friction in disturbing the stability of the rings. He introduced a statistical argument to calculate the probability of a molecule travelling a given distance without collision [1, pp. 25]. His description of a physical process by a statistical function was a major innovation in physics and led essentially to the statistical nature of the second law of thermodynamics and to speculations about the limitations of statistical knowledge. He thus tried to determine the speed with which the fragrance fills the air when a perfume-bottle is opened [9, 10]. In order to address this problem he derived the Maxwell distribution for molecular velocities. The distribution turned out to have the well-known bell-shaped curve now known as the normal distribution. This probability distribution is generally written as $\frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]$ where μ is the mean of the distribution and σ^2 is the variance. The most common application of this is in the field of statistical mechanics [http://en.wikipedia.org/wiki/Maxwell-Boltzmann_distribution]. The temperature of any (large) physical system is the result of the motions of the molecules and atoms which make up the system. These particles have a range of different velocities, and the velocity of any single particle constantly changes due to collisions with other particles. However, the fraction of a large number of particles within a particular velocity range is nearly constant. The Maxwell distribution of velocities specifies this fraction, for any velocity range, as a function of the temperature of the system. This distribution in the modern days is named after James Clerk Maxwell and Ludwig Edward Boltzmann and is called the Maxwell-Boltzmann distribution. Maxwell's discovery opened up an entirely new approach to physics, which led to statistical mechanics, to a proper understanding of thermodynamics, and to the use of probability distributions in quantum mechanics. In this Maxwell presented his distribution which is nowadays called the normal distribution. No one before Maxwell ever applied a statistical law to a physical process. He also predicted a new law of viscosity for gases in 1860, which was a magnificent piece of work, but was not devoid of flaws. His work inspired the inventor of statistical mechanics Ludwig Boltzmann (1844–1906), and their works led to the *Maxwell-Boltzmann* distribution of molecular energies.

In 1861 Maxwell published two parts [5, pp. 451-488] of his paper *On Physical Lines of Force* and parts 3 and 4 [5, pp. 489-513] the following year. In the first part he addressed that in all phenomena involving attractions or repulsions, or forces depending on the relative position of the bodies, we have to determine the magnitude and direction of the force which would act on a given body, if placed in a given position. The second part dealt with the question of: Is there any mechanical hypothesis as to the condition of the medium indicated by lines of force, by which the observed resultant forces may be accounted for? and, What is the mechanical cause of this difference of pressures produced by the lines of force?

In the third part of the paper he himself says: *I showed how the forces acting between magnets, electric currents, and matter capable of magnetic induction may be accounted for on the hypothesis of the magnetic field being occupied with innumerable vortices of revolving matter, their areas coinciding with the direction of the magnetic force at every point of the field. The centrifugal force of these vortices produces pressures distributed in such a way that the final* *effect is a force identical in direction and magnitude with that which we observe. In addition he described the mechanism by which these rotations may be made to coexist, and to be distributed according to the known laws of magnetic lines of force.*

I conceived the rotating matter to be substance of certain cells, divided from each other by cell-wells composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another. I have not attempted to explain this tangential action, but it is necessary to suppose in order to account for the transmission of rotation from the exterior to the interior parts of each cell, that the substances in the cells possess electricity of figure, similar in kind, though different in degree, that observed in solid bodies. The undulatory theory of light requires us to admit this kind of electricity in a luminiferous medium in order to account for transverse vibrations. We need not then be surprised if the magneto-electric medium posses the same property. In addition he introduced *the concept of displacement current in dielectrics force acting between two electric field bodies and the rate of propagation of transverse vibrations through the elastic medium of which the cells are composed, on the supposition that its elasticity is due entirely to forces acting between pairs of particles.* He obtains the velocity as 310, 740, 000 m/sec. Furthermore, he states that*: The velocity of transverse undulation in our hypothetical medium, calculated from the electromagnetic experiments of M. M. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical instruments of M. Fizeau that we can scarcely avoid the inference that light consist in the transverse undulation of the same medium which is the cause of electric and magnetic phenomenon.* He calculates the capacity of a Leyden jar with dielectric and observes that the inductive power of a dielectric varies directly as the square of the index of refraction and inversely as the magnetic inductive power. Finally, in the fourth part he establishes the connection between the distribution of lines of magnetic force and that of electric currents stating that it may be completely expressed by saying that the work done of a unit of imaginary magnetic matter when carried around a closed curve, is proportional to the quantity of electricity which passes through the closed curve.

Also in 1861, in setting up the standard for electrical resistance, he established the electrostatic units and the electromagnetic units to set up a coherent system of units and presented a thorough dimensional analysis. Thus he put dimensional analysis on a scientific footing which was discovered much earlier by Jean Baptiste Fourier and others. The ESU and the EMU system [9, 10] of units were later labeled as the Gaussian system of units, even though Gauss had nothing to do with it! In an 1863 memorandum he introduced the dimensional notation which were to become the standard of using the powers of Mass, Length and Time. He also showed that the relation between the two electromagnetic units, ESU and EMU, has a dimension LT^{-1}, which has a value very close to that of velocity of light [2, pp. 8]. Later on, he also made an experiment to evaluate this number.

This dimensional analysis prompted him to write the paper *A Dynamical Theory of Electromagnetic Field* [5, pp. 526-597] in 1864 where he set out *to establish the theory of the electromagnetic field because it has to do with the space in the neighborhood of the electric and magnetic bodies, and it may be called a Dynamical Theory, because it assumes that in that space there is matter in motion by which the electromagnetic phenomena are produced. In order to bring these results within the power of symbolical calculation, I then express them in the form of general equations of the electromagnetic fields. The equations express –*

(A) The relation between the electric displacement, true conduction and the total current compounded of both. This equation expresses the total current. The variation of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity which we may call p', q', r' to yield $p' = p + \frac{df}{dt}$; $q' = q + \frac{dg}{dt}$ and $r' = r + \frac{dh}{dt}$ where f, g, h denote the electric displacement along x, y, z.

(B) The relation between the lines of magnetic force and the inductive coefficients of a circuit, as already deduced from the laws of induction. This equation represents the equation of magnetic force and is given by: $\mu\alpha = \frac{dH}{dy} - \frac{dG}{dz}$; $\mu\beta = \frac{dF}{dz} - \frac{dH}{dx}$; $\mu\gamma = \frac{dG}{dx} - \frac{dF}{dy}$; where α, β, γ are the magnetic force acting on a unit magnetic pole and F, G, H are the electromagnetic momentum.

(C) The relation between the strength of a current and its magnetic effects, according to the electromagnetic system of measurements. This provides an equation for electric currents and expressed by $\frac{d\gamma}{dy} - \frac{d\beta}{dz} = 4\pi p'$; $\frac{d\alpha}{dz} - \frac{d\gamma}{dx} = 4\pi q'$; $\frac{d\beta}{dx} - \frac{d\alpha}{dy} = 4\pi r'$.

(D) The value of the electromotive force in a body, as arising from the motion of the body in the field, the alteration of the field itself, and the variation of electric potential from one part of the field to another. This results in the equation of the electromotive force given by P, Q, R on a moving conductor as: $P = \mu\left(\gamma\frac{dy}{dt} - \beta\frac{dz}{dt}\right) - \frac{dF}{dt} - \frac{d\psi}{dx}$; $Q = \mu\left(\alpha\frac{dz}{dt} - \gamma\frac{dx}{dt}\right) - \frac{dG}{dt} - \frac{d\psi}{dy}$; $R = \mu\left(\beta\frac{dx}{dt} - \alpha\frac{dy}{dt}\right) - \frac{dH}{dt} - \frac{d\psi}{dz}$.

(E) The relation between electric displacement, and the electromotive force which produces it. So when an electromotive force acts on a dielectric, it puts every part of the dielectric in a polarized condition, in which its opposite sides are oppositely electrified. If k is the ratio of the electromotive force to the electric displacement, then the equation for electric elasticity is given by $P = kf$; $Q = kg$; $R = kh$.

(F) The relation between an electric current, and the electromotive force which produces it. In isotropic substances if ρ is the specific resistance referred to unit of volume, then the equation for electric resistance is given by $P = -\rho p$; $Q = -\rho q$; $R = -\rho\gamma$.

(G) The relation between the amount of free electricity at any point, and the electric displacements in the neighborhood. If e is the quantity of free positive electricity contained in unit of volume at any point of the field, then the equation for free electricity is given by $e + \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0$.

(H) The relation between the increase or diminution of free electricity and the electric currents in the neighborhood. This gives rise to the equation of continuity $\frac{de}{dt} + \frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} = 0$.

Hence, there are twenty equations in all, involving twenty variable quantities summarized in Table 1 including the symbols used by Maxwell and the modern day notations.

TABLE 1. Twenty Variables Originally Introduced by Maxwell

Variable Name Used by Maxwell (Equivalent Modern Name)	Symbol Used by Maxwell	Modern Equivalent Vector/Scalar
Electromagnetic Momentum (Magnetic Vector Potential)	F, G, H	$\boldsymbol{A}$
Magnetic Force (Magnetic Field Intensity)	α, β, γ	$\boldsymbol{H}$
Electromotive Force (Electric Field Intensity)	P, Q, R	$\boldsymbol{E}$
Current Due to True Conduction (Conduction Current Density)	p, q, r	$\boldsymbol{J}$
Electric Displacement (Electric Flux Density)	f, g, h	$\boldsymbol{D}$
Total Current (p^1, q^1, r^1)Including Variation of Displacement (Conduction plus Displacement Current Density)	$\left\{\begin{array}{l} p^1 = p + \frac{\mathrm{d}f}{\mathrm{d}t} \\ q^1 = q + \frac{\mathrm{d}g}{\mathrm{d}t} \\ r^1 = r + \frac{\mathrm{d}h}{\mathrm{d}t} \end{array}\right\}$	$\boldsymbol{J}_T$
Quantity of Free Electricity (Volume Density of Electric Charge)	e	ρ
Electric Potential (Electric Scalar Potential)	ψ	ψ

He also derived the coefficients of induction between two circuits in this paper. As he stated in the abstract of his paper: *The proposed theory seeks for the origin of electromagnetic effects in the medium surrounding the electric or magnetic bodies, and assumes that they act on each other not immediately at a distance, but through the intervention of a medium which is progressive in nature*. It is also important to note that Maxwell believed in the aether theory as next he states: *The existence of the medium is assumed as probable, since the investigations of Optics have led philosophers to believe that in such a medium the propagation of light takes place*. He stated that the following quantities: *Electric currents by conduction, electric displacements, and Total currents; Magnetic forces, Electromotive forces, and Electromagnetic Momenta* are involved in the mathematical expression for the electromagnetic fields. *Each of these quantities being directed quantity has three components; and beside these we have two others, the Free electricity and the Electric potential, making twenty quantity in all. There are twenty equations between these quantities, namely Equations of Total Currents, of Magnetic Force, of Electric Currents, of Electromotive Force, of Electric Elasticity, and of Electric Resistance, making six sets of three equations, together with one equation of Free Electricity, and another of Electric Continuity. The equations show that the transverse disturbances, and transverse disturbances only, will be propagated through the field, and that the number which expresses the number of electrostatic units of electricity in one electromagnetic unit, the standards of space and time being the same. The first of these results agrees with the undulatory theory of light as deduced from optical experiments. The second may be judged of by a comparison of the electromagnetical experiments of Wilhelm Eduard Weber and Rudolf*

Hermann Arndt Kohlrausch with the velocity of light as determined by astronomers in heavenly spaces, and by M. Foucault in the air of his laboratory.

Electrostatic units in an Electromagnetic Unit: 310,740,000 meters/second
Velocity of light as found by Armand Hippolyte Louis Fizeau : 314,858,000 meters/second
Velocity of light as found by Jean Bernard Leon Foucault : 298,000,000 meters/second
Velocity of light deduced from aberration : 308,000,000 meters/second

At the outset of the paper, the dynamical theory of the electromagnetic field borrowed from the undulatory theory of light the use of its luminiferous medium. It now restores the medium, after having tested its powers of transmitting undulations, and the character of those undulations, and certifies that the vibrations are transverse, and that the velocity is that of light. With regard to normal vibrations, the electromagnetic theory does not allow of their transmission. What then is light according to electromagnetic theory? It consists of alternate and opposite rapidly recurring transverse magnetic disturbances, accompanied with electric displacements, the direction of the electric displacement being at right angles to the magnetic disturbance, and both at right angles to the direction of the ray. The theory does not attempt to give a mechanical explanation of the nature of magnetic disturbance or of electric displacement, it only asserts the identity of these phenomena..... It discloses a relation between the inductive capacity of a dielectric and its index of refraction. .. The propagation of vibrations in a conducting medium is then considered and it is shown that the light is absorbed at a rate depending on the conducting power of the medium.

To answer the question in more details as to how did Maxwell reached the definite conclusion that light was electromagnetic in nature and at which point did it cease to be speculation. The first mention of the great discovery comes in a letter which he wrote to Faraday on the date 19th October, 1861 [2, pp. 683-688, 4, p. 102]. For this part, we refer to his book which was first published in 1873 and in the revised form in the second edition after his death in 1881 [7]. In [7] Maxwell first presents the 20 equations as mentioned earlier. Even though he talks about the usage of the Quaternion convention, the final expressions are in scalar form [7, Ch. IX]. In Ch. X he introduces the two systems of units, ESU and EMU and illustrates that if the units of length, mass and time are the same in the two systems, the number of electrostatic units of electricity contained in one electromagnetic unit is numerically equal to a certain velocity, the absolute value of which does not depend on the magnitude of the fundamental units employed [7, Part II, p. 245]. In Ch. XX he shows that the disturbances in a media are propagated through a transverse electromagnetic wave which propagates at a velocity close to that of light. He furthermore compares the velocity of light measured by Fizeau (314000000 m/sec), that measured through aberrations and sun's parallax (308000000 m/sec) and by Foucault (298360000 m/sec) and that measured by Weber and Kohlrausch from the ratio between the ESU and the EMU units (310740000 m/sec) and by Maxwell (288000000 m/sec) and Lord Kelvin (282000000 m/sec). From this comparison Maxwell asserts that the velocity of light and the ratio of the two units are quantities of the same order of magnitude. Next, Maxwell observes in his theory that the refractive index of the media is related to the square root of the dielectric constant of a transparent media. He considers melted paraffin. The dielectric constant of paraffin has been measured by J. C. Gibson and T. Barclay to be 1.975 [7, p. 398] and the refractive index of the same material has also been measured by Dr. J. H. Gladstone to be 1.43 [7, p. 398] . In this way he relates the electrical properties of a medium to that of optical properties of the medium. Finally, in Ch. XXII he explains the properties of ferromagnetism and diamagnetism by molecular currents and tries to explain the Faraday effect of the rotation of the polarization of the light passing through a magnetic medium and thus relating the magnetic properties to the optical properties. In this way, his Treatise [8] illustrates that light is electromagnetic in nature, even though he lacked the information about the boundary conditions to solve the wave equation completely!

He liked to use the Quaternion convention. However, he wrote the final equations in the scalar form even though he used the terms “curl”, “convergence” and “gradient”. Nowadays, convergence is replaced by its negative, which is called divergence, and the other two are still in the standard mathematical literature. These are available in his paper *On the Mathematical Classification of Physical Quantities* [6, pp. 257-266]. Maxwell identified light with electromagnetic waves and introduced the concept of aether as the basic medium of the electromagnetic field to retain the possibility of a mechanical interpretation. He presented his first paper on the new theory before the Royal Society in 1864 and published the comprehensive *Treatise on Electricity and Magnetism* in 1873.

He was also the first to suggest using a centrifuge to separate gases, which is still being used in modern times and published in *Instruments connected with fluids* [6, pp. 523-528, 9, 10]. He also showed that radiation pressure from the sun exists, which has a mean pressure of 8.82×10^{-8} pounds per square foot [7, Pt. II, pp. 402]. He went on to say that a body exposed to sunlight would experience this pressure on the illuminated side only, and would therefore be repelled from the side on which the light falls. This was later used to explain why the tail of a comet moves away from the sun.

In 1866, he wrote *Dynamical Theory of Gases* [6, pp. 26-78] and produced the first statistical law of physics. He wrote a paper on Boltzmann’s theorem in 1879 on the *Average Distribution of Energy* in a system of material points. This enabled people to explain the properties of matter in terms of the behavior, en masse, of its molecules. One of the ideas in the paper was the method of ensemble averaging, where the whole system is much easier to analyze, rather than dealing with individual components. Interestingly, this mode of analysis is quite prevalent nowadays in most signal processing and communications theory applications. However, his result for the specific heat of air was off from the measurement. Instead of trying to explain this discrepancy in his theory by ingenious attempts [9, 10], he said: *Something essential to the complete statement of the physical theory of molecular encounters must have hitherto escaped us, and that the only thing to do was to adopt the attitude of thoroughly conscious ignorance that is the prelude to every real advance in science*. He was right. The explanation came 50 years later from quantum theory.

When creating his standard for electrical resistance, he wanted to design a governor to keep a coil spinning at a constant rate. He made the system stable by using the idea of negative feedback. He worked out the conditions of stability under various feedback arrangements in his paper *On Governors* [6, pp. 105-120] in 1868. This was the first mathematical analysis of control systems. He showed for the first time that for stability the characteristic equation of the linear differential equation has to have all its roots with negative real parts. This work did not get any attention till 1940, when gun control radars were in demand during the Second World War. After the war, Norbert Wiener (1894–1964) took things further and developed the science of cybernetics, based on his paper. He also produced the first standard of electrical resistance in 1868.

In 1868, *On a Method of Making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light* [6, pp. 125-143], he measured the speed of light by using 2,600 batteries to produce 3,000 volts [9, 10]. The goal was to balance the electrostatic attraction between two charged metal plates against the magnetic repulsion between two current carrying coils and built a balance arm to do this. He got a result of 288,000 km/sec as compared to the current accepted value of 299,792.5 km/sec.

In 1871, Maxwell showed how a circuit containing both capacitance and inductance would respond when connected to generators containing alternating currents of different frequencies. He developed the phenomenon of electrical resonance in parallel to acoustic resonance developed by Sir John William Strutt, Lord Rayleigh and also suggested naming Rayleigh's book on acoustics *Theory of Sound*. Maxwell developed the electrical analog when reviewing Rayleigh's paper and wrote about it to Rayleigh [2, pp. 598-608]. Maxwell provided a simpler mathematical expression for the wave velocity and group velocity again when reviewing a paper by Rayleigh *On Progressive Waves* [3, pp. 547-549].

Sir John Ambrose Fleming wrote [4, p.118]: *In electricity course, he gave us a new and powerful method of dealing with problems in networks and linear conductors. Kirchoff's corollaries of Ohm's Law had provided a means only applicable in the case of simple problems in which one could foresee the direction of flow of current in each conductor. But that was not possible in complicated networks. Maxwell initiated a new method by considering the actual current in each wire to be the difference of two imaginary currents circulating in the same direction round each mesh of the network. In this way, the difficulty of foreseeing the direction of the real current was eliminated. The solution of the problem was then reduced to the solution of a set of linear equations and the current in any wire could be expressed as the quotient of two determinants. After Maxwell's death, in 1885 I communicated a paper to the Physical Society of London in which the method was extended so as to give an expression for the electrical resistance of any network between any two points*. One would immediately recognize this as a method for writing the loop equations that are currently available in all undergraduate electrical engineering textbooks dealing with electrical circuits, and yet no mention is made of Maxwell, the inventor of this technique! The point here is that Kirchoff no doubt wrote the theorems on the loop and the node equations but did not provide a methodology on how to solve for them in an arbitrary circuit. That job was completed by Maxwell.

He, along with the English biologist Thomas Henry Huxley (1825–1895), was the joint scientific editor of the 9th edition of Encyclopedia Britannica in 1879. There he provided an account of the motion of earth through aether. He had done experiments in 1860 which failed to detect any effect, however, he strongly believed in the existence of aether. At that time his paper was rejected by Lord Kelvin, who was the reviewer! Maxwell suggested that aether could perhaps be detected by measuring the velocity of light when light was propagated in opposite directions. He had further discussions in a letter to David Peck Todd, an astronomer at Yale. Maxwell's suggestion of a double track arrangement led A. A. Mickelson, when he was working under Helmholtz as a student, to undertake his famous experiments later on aether drag in the 1880s and the rest is history.

Maxwell always delivered scientific lectures for the common people using models. He also was very prolific in writing limericks, as we will see.

He wrote a book on The *Theory of Heat* in 1871 and provided a completely new formulation of the relationships between pressure, volume, temperature, and entropy and expressed them through differential equations known as *Maxwell's relations* (http://en.wikipedia.org/wiki/Maxwell's_relations). He introduced the "Maxwell demon", as termed by Lord Kelvin, i.e., the molecule-sized creature which was going to defy the second law of thermodynamics. The goal was that this demon can separate low velocity molecules from high velocity ones by opening an aperture and, thus, making heat flow from a colder region to a hotter one defying the second law of thermodynamics. Hence, the demon is generating a perpetual motion machine: the machine will keep on working till the temperature difference between the two regions fell back to zero and then we would be back to where we started. This cannot really happen. Using these concepts Leó Szilárd (1898–1964) in 1929 showed that the very act of acquiring information about a system increases its entropy proportional to the amount of information gathered. Through such work of Szilard (while working with Einstein on the development of a home refrigerator without moving parts, applying for the patent of the cyclotron the same year, and being the chief investigator of the Manhattan project) and others, Maxwell's demon helped the creation of information theory, now an essential part of communication and computing. These basic concepts of information theory were reinvented by Claude Elwood Shannon 20 years later and extended it to a different level establishing the basis of coding theory!

He also wrote a paper *On Hills and Dales* in 1870 [6, pp. 233-240], explaining that the surface of the Earth has high areas or hills and low areas with a bottom point. There are also ridges, valleys, or dales, and passes. He showed that the numbers of each of these features are somehow related by mathematical rules. His original ideas about the Earth's surface have now evolved into a branch of topology called global analysis.

Maxwell also wrote a paper *On the Protection of Buildings from Lightning* [3, pp. 355-357, 6, pp. 538-540]. In that paper he pointed out that most of the published writing dealt with are on the necessity of obtaining what Telegraph engineers call a good earth connection. The telegraphist uses the earth to complete his circuit, therefore it is of great importance to him; but the protection of buildings from electric discharges has a different aim and a different method.

Even though Maxwell has influenced development in many areas of physical sciences and had started a revolution in the way physicists look at the world, he is not very well known, unfortunately, outside some selected scientific communities. In fact, when the Royal Society of London held its tercentenary celebration, Queen Elizabeth II presided and praised a number of former Fellows – presumably listed by the Society. Inexplicably, Maxwell was not among them [9, 10]. He has been more widely commemorated elsewhere, even in countries without a strong scientific tradition. For example, the governments of Mexico, Nicaragua, and San Marino are among those who have issued postage stamps in his honor [9, 10]. Some claim that the reason Maxwell (He was Scottish!) was not so well known (he was not even knighted) is because he was too humble and never proselyte his theory. There may be some truth to that. In addition, there may be other reasons. Sir Joseph John Thomson [4, p. 10], speaking at the centenary celebration of Maxwell, said that, according to his teacher, William Hopkins (1793–1866) at Cambridge, Maxwell was unquestionably the most extraordinary man he had met with in the whole course of his experience; that it appeared impossible for Maxwell to think wrongly on any physical subject, but that in analysis he was far more deficient. … The public lectures were however read and he was compelled by the manuscript to keep to the track.

In the same celebration, Sir James Jeans succinctly summarized his contributions [4, p. 93]: *as of purely abstract in nature. As a consequence, Maxwell did not alter the face of civilization as Faraday did, or at least did not alter it so immediately or in a manner obvious to the eye. It could hardly have been written of him during his lifetime, as it was of Faraday, that our life is full of resources which are the results of his labours; we may see at every turn some proof of the great grasp of his imaginative intellect. Faraday had used his clear vision and consummate skill as an experimenter to explore those strata of nature which lie immediately* *under our hands; Maxwell used his clear vision and consummate skill as a theorist to explore the deeper strata in which the phenomenon of the upper strata have their origin.*

However, as Sir James Jeans pointed out [4]: *It was not keeping with Maxwell's methods that he should finish off a piece of work so completely that nothing could be added to it, his plan was rather to open up wide vistas which could provide work in their detailed exploration for the whole generations yet to come. ... It was his power of profound physical intuition coupled with adequate, although not outstanding mathematical technique, that lay at the basis of Maxwell's greatness.*

Sir Horace Lamb [4] said: *He had his full share of misfortunes with the blackboard, and one gathered the impression, which is confirmed I think by the study of his writings, that though he had a firm grasp of essentials and could formulate great mathematical conceptions, he was not very expert in the details of minute calculations. His physical instincts saved him from really vital errors.*

However, there may be an additional reason that he was not recognized during his life time for his work, which is that he had an eccentric side to his personality. He had a tendency to make insulting remarks, largely in the form of sardonic limericks! During the 1874 British Association meeting at Belfast he delivered several poems, in one of which he refers to members of this highly respected body in this way [11, p. 637]:

So we who sat, oppressed with science,
As British asses wise and grave,
Are now transformed to Wild Red Lions…

(The Red Lions are a club formed by members of the British association to meet for relaxation after the graver labors of the day).

And after hearing a lecture by the Scottish physicist Peter Guthrie Tait (1831–1901) in 1876, he wrote [11, p.646]:

Ye British asses, who expect to hear
Ever some new thing,
I've nothing to tell but what, I fear,

May be a true thing.
For Tait comes with his plummet and his line,

Quick to detect your
Old bosh new dressed in what you call a fine
Popular lecture.

Also in his private letters primarily to Lord Rayleigh, Maxwell used to refer to the British Association *as Brit. Ass.,* perhaps for abbreviation but sure looks …..!

And again in 1878 where he devoted the Rede lecture on the invention of the telephone as:

One great beauty of Professor Bell's invention is that the instruments at the two ends of the line are precisely alike…The perfect symmetry of the whole apparatus – the wire in the middle, the two telephones at the ends of the wire; and the two gossips at the ends of the telephones may be very fascinating to a mere mathematician, but would not satisfy the evolutionist of the Spencerian type, who would consider anything with both ends alike, to be an organism of the *very low type, which must have its functions differentiated before any satisfactory integration can take place.*

It is rather amusing to note that in recent times this statement of Maxwell in many places has been used to claim that Maxwell was against the evolutionary theory of Charles Robert Darwin, which the British philosopher and sociologist Herbert Spencer (1820–1903) popularized. However, according to Fleming, who attended the lecture: *It was a brilliant discourse, illustrated by flashes of wit, apt analogies and much learning but it was not of the type most useful to convey to unscientific hearers an idea of the mode in which a telephone operates*.

Maxwell's personality often showed two additional unusual characteristics. The first is that he could return to a subject, often after a gap of several years, and take it to new heights using an entirely new approach. He did this twice with electromagnetism. The second is that often his intuition led him to correct results even when he had made mistakes along the way. He was tolerant of the mistakes of others, but was very critical of the failure to be honest and open to the readers. He rebuked Poisson for telling lies about the way people make barometers and Ampère for describing only perfected experiments to demonstrate his law of force and hiding the poor experiments by which he had originally discovered the law [9, 10].

4. What was/is Maxwell's Electromagnetic Theory?

Interestingly there is no clear cut answer to this question as we will see. This section is derived from the writings of several scientists who themselves modified his theory. First, we describe what Maxwell presented, followed by the modifications made by others. As elucidated by Sir James Jeans [4]:

Maxwell pictured electromagnetic theory of light in terms of a medium whose properties could be specified completely in terms of a single mathematical constant. He saw that if the value of this constant could once be discovered, it ought to become possible to predict all phenomena of optical theory with complete mathematical precision. Maxwell showed that the constant in question ought to be merely the ratio of electromagnetic and electrostatic units of electricity and his first calculations suggested that this was in actual fact equal to the constant of the medium which measured the velocity of light.

The first mention of the great discovery comes in a letter, which he wrote to Michael Faraday on 19th October 1861: I suppose the elasticity of a sphere to react on the electrical matter surrounding it, and press it downwards. From the determination by Kohlrausch and Weber of the numerical relation between static and the magnetic effects of electricity, I have determined the elasticity of the medium in air, and assuming that it is the same with the luminiferous aether, I have determined the velocity of propagation of transverse vibrations. The result is 193,088 miles per second. Fizeau has determined the velocity of light as 193,118 miles per second by direct experiment.

Even though the two numbers quoted above agreed to within 30 miles per second, oddly enough both are in error by more than 6,000 miles a second. When Maxwell came to publish his paper, A Dynamical Theory of the Electromagnetic Field*, probably the most far reaching paper he ever wrote, he gave the two velocities in terms of kilometers per second: Kohlrausch and Weber (310,740,000 meters a second) and Fizeau (314,858,000 meters a second) and is nowhere near the figure. Happily he seems to have realized that the velocity of light was not at all accurately known, and so he did not allow himself to be deterred, as Newton had been, by a substantial numerical disagreement in his law of universal gravitation.*

According to English physicist Sir Oliver Joseph Lodge (1851–1940) [4]:

Maxwell perceived that a magnetic field was wrapped around a current, and that a current could equally well be wrapped around a magnetic field, that in fact the relation between that was reciprocal, and could be expressed mathematically by what he subsequently called curl. *When the two fields coexisted in space, the reaction between them could be expressed as the curl of a curl, and this he simplified down to the well known wave equation, the velocity of propagation of the wave being the reciprocal of the geometric mean of an electric and a magnetic constant. This velocity for a time he called the number of electrostatic units in a magnetic unit, and proceeded to devise experiments whereby it could be measured. Experiments made by him at King's College resulted in some near approach to the velocity of light, so that thenceforth, in his mind light became an electromagnetic phenomenon. He was assisted in his ideas by an imaginative constructive model of the aether, a model with rolling wheels and sliding particles, which subsequently he dropped, presumably as being too complex for reality and remained satisfied with his more abstract equations, which were reproduced in his great* Treatise *in 1873.*

This provides one reason why Maxwell's theory was so difficult to follow. By never identifying his physical pictures with reality, Maxwell left himself free to discard one picture and adopt another as often as expediency or convenience demanded. He described his method of procedure in the following words:

If we adopt a physical hypothesis, we see the phenomenon only through a medium, and are liable to that blindness to facts and rashness in assumption which a partial explanation encourages. We must therefore discover some method of investigation which allows the mind at every step to lay hold of a clear physical conception, without being committed to any theory founded on physical science for which that conception is borrowed.

Maxwell considered Faraday's line of force similar to the lines of flow of a liquid. Maxwell himself wrote about it to Lord Kelvin [13]:

I suppose that the "magnetic medium" is divided into small portions or cells, the divisions or cell walls being composed of a single stratum of spherical particles these particles being "electricity". The substance of the cells I suppose to be highly elastic both with respect to compression and distortion and I suppose the connection between the cells and the particles in the cell walls to be such that there is perfect rolling without slipping between them and that they act on each other tangentially. I then find that if the cells are set in rotation, the medium exerts a stress equivalent to a hydrostatic pressure combined with a longitudinal tension along the lines of axes of rotation. ... Thus there will be a displacement of particles proportional to the electro-motive force, and when this force is removed, the particle will recover from displacement. I have calculated the relation between the force and the displacement on the supposition that the cells are spherical and that cubic and linear elasticities are connected as in a perfect solid. I have found from this the attraction between two bodies having given free electricity on their surfaces. And then by comparison with Weber's value of the statical measure of a unit of electrical current I have deduced the relation between elasticity and density of the cells. The velocity of the transverse undulations follows from this directly and is equal to 193,088 miles per second, very nearly that for light.

Even though the final results of the original Maxwell's theory are valid even today, however, the intermediary steps used to arrive at the conclusion was in question [13]. The first problem was associated with the definition of the charge. For example, consider a positively charged ball that is placed inside an infinite dielectric medium. Modern theory defines [13, p.25] the positive surface charge on the surface of the conductor as seen in Figure 5a. This charge creates an electric field which produces polarization throughout the dielectric. Next we divide the dielectric using an imaginary surface *C*. One part (*A*) of the dielectric lies between the conducting sphere and *C*; the other part (*B*) lies between *C* and goes to infinity. The innermost boundary of *A* which actually touches the sphere, according to modern theory will carry a negative polarization charge which is smaller in magnitude than the conduction charge on the sphere. The outermost boundary of *A*, characterized by the surface *C*, bears a positive polarization charge numerically equal to the negative polarization charge on the inner surface of *A*. The charge on the outermost boundary of *A* is exactly compensated by a negative polarization charge on the innermost boundary of *B*, i.e., by a charge on surface *C* considered as the inner boundary of *B*. Accordingly no space charge at all exists and we have only the positive conduction charge and the numerically smaller negative polarization charge on the surface of the dielectric which is immediately adjacent to it as shown in Figure 5a [13, p.26]. Now if we use Maxwell's interpretation of the same situation using his quote: *The charge therefore at the bounding surface of a conductor and the surrounding dielectric; which in the old theory was called the charge of the conductor, must be called in the theory of induction (Maxwell's theory) the surface charge of the surrounding dielectric* [7, Vol. I, Art 111]. This situation is illustrated in Figure 5b.

In Maxwell's theory [13, p.27], we begin with a displacement ***D*** which exists throughout the dielectric and which points away from the center of the sphere. Since the displacement points away from the center of the sphere, it enters *B*'s inner boundary in a direction parallel to that boundary's inward-directed normal. According to Maxwell's definition of charge, the inner boundary of *B* has a positive charge on it numerically equal to ***D*** as shown in Figure 5b. In addition, the outermost boundary of *A* (surface *C*) coincides with the innermost boundary of *B*. But the displacement exist from the outermost boundary of *A* in a direction opposite to the inward directed normal of that boundary. Therefore, the outermost boundary of *A* has on it a negative charge equal and opposite to the positive charge on the inner boundary of *B* as shown in Figure 5b. Since the boundaries coincide, no net charge can exist anywhere in the dielectric. At the surface of the sphere the situation is different and for that we need to look at first the inner boundary of the dielectric, which touches the sphere and secondly, the surface of the sphere itself. The displacement enters the dielectric boundary parallel to its inwardly directed normal and so we have on this surface a positive charge. But since no displacement at all exists within the sphere, its surface is uncharged. Consequently, the positive charge on the inner surface of the dielectric is uncompensated. The result is that what modern theory calls the positive surface charge of the conductor, Maxwell's theory called the positive surface charge of the inner surface of the dielectric [13, p.27]. This explanation raises two questions which are: Is this consistent with charge conservation and the Coulomb's law? and How do we explain the existence of discontinuity in displacement without a source? As described in [13], the great difficulty in the Maxwell theory in answering these two questions was later addressed by Hertz and Heaviside.

Maxwell considered magnetism as a phenomenon of rotation and electric currents as a phenomenon of translation. So in any magnetic field the medium is in rotation about the lines of magnetic force. Maxwell's theory sought unity through a set of field equations coupled to

Hamilton's principle, named after the Irish mathematician Sir William Rowan Hamilton (1805–1865). In current bearing linear circuits, Maxwell thought that the currents were linked by rigid constraints to an intervening medium called the aether. Thus, the most difficult concepts for the modern reader to grasp in the original Maxwell's theory are the concepts of "charge" and "current". In modern theory, charge is the source of the electric field and current is the source of the magnetic field. In his theory, charge is produced by the electric field; current, in the usual sense, is the rate of change of charge over time, and is only indirectly related to the magnetic field. Therefore, in Maxwellian theory, charge is a discontinuity of the displacement $\boldsymbol{D}$ and not in $\boldsymbol{E}$. Maxwell's goal was to create a theory of electromagnetism which made no use whatsoever of the microstructure of the matter. To Maxwell, the conduction current was effectively a continuous series of charging and discharging. The conduction current then is the process and growth of displacement [13, p.29]. Maxwell proposed all currents are closed. In this fashion, he introduced the displacement current. Maxwell quite explicitly limited electric polarization to the boundary conditions on the flux characteristics of electric displacement and magnetic induction.

In the European continent, physicists of the period were intimately familiar with field equations. However, each of them attempted to obtain Maxwell's field equations as a limiting case of Helmholtz's (entirely non Maxwellian) polarization theory of aether and matter. Maxwell's theory was based on a property of vortex notion, which Helmholtz had deduced (1858). Even those who no longer reached the Maxwell equations via Helmholtz (like Hendrik Antoon Lorentz after 1892 or Hertz after 1890) continued to bear unmistakable marks of the Helmholtz polarization theory. However, there is a marked distinction between the two theories due to the difference between those who viewed electricity as a by-product of the field processes from those who did not [13]. Helmholtz's theory makes a distinction between conduction and polarization charge, with free charge being their sum. Helmholtz's theory consisted of three components [13, p.184]:

1. expression for electromagnetic potentials and the forces derived from them
2. a continuity equation linking charge and current, and
3. a model for an electrically and magnetically polarizable medium.

The conflict between Helmholtz's and Maxwell's theories occurs where one would expect to find it, i.e., in the continuity equation. In Helmholtz's theory, all fields involve interaction between charge densities, and these interactions are not in fact propagated [13, pp.184-186]. Only the polarizations propagate, charge interactions are always instantaneous. The basic difference is that in Maxwell's theory charge is the discontinuity in the electric displacement whereas in Helmholtz's theory it is with the discontinuity in the electric field. Thus, the problem with Helmholtz's theory is then at material interfaces where there will be charges as the electric field is discontinuous and where the displacement is not. Jules Henri Poincaré (1854–1912) [13, pp. 187-193], the French mathematician, theoretical scientist, and philosopher of science, said on Maxwell's *Treatise* that was published in 1873: *I understand everything in this book except what is meant by a body charged with electricity*.

In summary, Maxwell's *Treatise* provided an immense fertile theory, but in a form which was awkward, confusing, and on some points deviated widely from the modern theory [18]. It was especially ill-tuned to handling the propagation problems that were coming into increasing prominence in the 1880s in connection with advances in telegraphy, telephony, and the study of electromagnetic waves. Before Maxwell's theory could gain wide acceptance and come into general use, it required substantial revision and clarification; both its physical principles and their mathematical expressions had to be put into a simpler and more easily grasped form. The most important steps in this process were taken in the mid 1880s by John Henry Poynting, George Francis FitzGerald, and Oliver Heaviside on the flow of energy for an electromagnetic field [18]. We now look at this development in more details.

Maxwell was able to show [7, Art. 783 & 784; Equations 8 & 9] that in free space $(d^2J)/(d\,t^2)+\{d(\nabla^2\Psi)\}/(d\,t)=0$, where J is the conduction current density and ψ is the scalar electric potential. He then made an important assertion, for which he provided no real justification: "$\nabla^2\Psi$ ", he said, *which is proportional to the volume density of the free electricity, is independent of t*, i.e., he claimed that the electric potential is determined solely by the spatial distribution of charge which in a non-conductor does not change. This is the assumption usually made in electrostatics, and Maxwell simply extended it to general electromagnetic theory without alteration or explanation. Time independence implied that the electric potential adjusted instantaneously across all space to any changes in the positions or magnitudes of the charges. It also implied $(d^2J)/(d\,t^2)=0$, so that as Maxwell wrote *J must be a linear function of t, or a constant, or zero and we may therefore leave J and ψ out of account in considering wave disturbances*. In practice, Maxwell generally took $J = 0$ and so worked out what we now call the *Coulomb Gauge*, a gauge well suited to electrostatic problems but with the serious drawback in treating changing fields that it requires the electric potential to be propagated instantaneously [18]. FitzGerald differed from Maxwell on this point and instead of assuming the two potentials to be independent as Maxwell did, FitzGerald put $J=-(d\Psi)/(d\,t)$ or equivalently: $\nabla\bullet\boldsymbol{A}+(d\Psi)/(d\,t)=0$. This *Lorenz Gauge*, as it was later called, is much better suited to treating propagation phenomenon than was Maxwell's *Coulomb Gauge* with $J = 0$. This new gauge thus eliminated the question of the instantaneous propagation of the electric potential. However, FitzGerald found that Heaviside had independently done it already [13]! As FitzGerald and Rowland put it in 1888, *That ψ should be murdered from treating propagation problems* [18]. Soon after, the same fate happened to ***A***, as Heaviside puts it, *not merely the murder of Maxwell's ψ, but of that wonderful three legged monster with a scalar parasite on its back, the so called electrokinetic momentum at a point – that is the vector potential itself* [18]. That is the first complete modification of Maxwell's theory done by Heaviside and Hertz to get rid of the potentials and to start the problem with the sources, i.e., currents and charges.

Poynting in 1883 showed how the energy from an electric current passes from point to point, i.e., by what paths and according to what law does it travel from the part of the circuit where it is first recognizable as electric and magnetic to the parts where it changed into heat and other forms. In addition, Heaviside in 1884–1885 cast the long list of equations that Maxwell had given in his *Treatise* into the compact and symmetrical set of four vector equations now universally known as *Maxwell's equations*. Heaviside independently developed Poynting's theorem six months later. Reformulated in this way, Maxwell's theory became a powerful and efficient tool for the treatment of propagation problems, and it was in this new form (*Maxwell Redressed*, as Heaviside called it) that the theory eventually passed into general circulation in the 1890s. Heaviside's distance from the mainstream of British mathematical physics made it easier for him to dispense with the potentials and the Lagrangian methods favored by members of the Cambridge school and to approach the problem from a new and, as he believed through his vector operational calculus, into more fruitful directions [18].

After Maxwell's books on the treatise were published in the European Continent, Helmholtz has been trying to understand Maxwell's theory of electromagnetism and to compare it with a theory based mostly on Newtonian mechanics [19]. In 1879 Helmholtz called for an experimental validation of Maxwell's theory and had it published as a prize problem of the Prussian Academy of Science, often referred to as the *Berlin prize*. For the 1882 prize the problem stated [19]: *The theory of electrodynamics which was brought forth by Faraday and was mathematically executed by Mr. Cl. Maxwell presupposed that the formation and disappearance of the dielectric polarization in insulating media – as well as in space – is a process that has the same electrodynamic effects as an electrical current and that this process, just like a current, can be excited by electrodynamically induced forces. According to that theory, the intensity of the mentioned currents would have to be assumed equal to the intensity of the current that charges the contact surfaces of the conductor. The Academy demands that decisive experimental proof be supplied either*

for or against the existence of electrodynamic effects of forming or disappearing dielectric polarization in the intensity as assumed by Maxwell or

for or against the excitation of dielectric polarization in insulating media by magnetically or electrodynamically induced electromotive forces.

Answers to these questions have to be submitted by March 1, 1882 and the prize of 955 marks will be awarded. At that time Hertz was working with Helmholtz at the Physical Institute in Berlin. Helmholtz thought that one of his students, Heinrich Hertz, would be most likely to succeed in this experimentation. He suggested to Hertz that, should he address this problem, the resources of the institute will be available to him. However, at that time Hertz gave up the idea because he thought a solution was not possible, as he found no adequate sources for generation of high frequencies. However, he continued thinking about this problem. Hertz did an analytical thesis on the induced currents in a rotating metal sphere in a magnetic field [19]. After graduating in 1880, he stayed on as an assistant to Helmholtz and then went to the University of Kiel as an instructor of theoretical physics. At Kiel, Hertz had no laboratory and was very impatient working only in theoretical physics. In 1884 at Kiel, he published the paper *On the relation between Maxwell's fundamental electromagnetic equations and the fundamental equations of the opposing electromagnetics*. He concluded that if he had to make a choice, he would choose Maxwell's theory. This work convinced him to carry out experimental work to verify this theory. In his book, he himself addresses the question as to what exactly is Maxwell's theory. In his words [17, 13, p.191]:

Maxwell left us as the result of his mature thought a great treatise on Electricity and Magnetism; it might therefore be said that Maxwell's theory is the one propounded in that work. But such an answer will scarcely be regarded as satisfactory by all scientific men who have considered the question closely. Many a man has thrown himself with zeal into the study of Maxwell's work, and even when he has not stumbled upon unwanted mathematical difficulties, has never the less been compelled to abandon the hope of forming for himself an altogether consistent view of Maxwell's ideas. I have fared no better myself. Notwithstanding the greatest admiration for Maxwell's mathematical conceptions, I have not always felt quite certain of having grasped the physical significance of the statements. Hence, it was not possible for me to be guided directly by Maxwell's book. I have rather been guided by Helmholtz's work, as indeed may plainly be seen from the manner in which the experiments are set forth. But unfortunately, in the special limiting case of Helmholtz's theory which lead to Maxwell's equations, and to which the experiments pointed, the physical basis of Helmholtz's theory disappears, and indeed it does, as soon as action-at-a-distance is disregarded. I therefore endeavored to form for myself in a consistent manner the necessary physical conceptions, starting from Maxwell's equations, but otherwise *simplifying Maxwell's theory as far as possible by eliminating or simply leaving out of consideration those portions which could be dispensed within as much as they could not affect any possible phenomena… To the question, "What is Maxwell's Theory?" I know of no shorter or more definite answer than the following: - Maxwell's theory is Maxwell's system of equations. Every theory which leads to the same system of equations, and therefore comprises the same possible phenomena, I would consider as being a form or special case of Maxwell's theory; every theory which leads to different equations, and therefore to different possible phenomena, is a different theory. Hence, in this sense, and in this sense only, may the two theoretical dissertations in the present volume be regarded as representations of Maxwell's theory. In no sense can they claim to be precise rendering of Maxwell's ideas. On the contrary, it is doubtful whether Maxwell, were he alive, would acknowledge them as representing his own views in all respects.*

The 1884 paper helped him to get his next appointment at the Technical High School at Karlsruhe in 1885, where he started his experimental work and discovered how to generate and detect electromagnetic energy. Finally, in 1887 he provided experimental results for the Berlin Prize and it was published in his 1888 paper. It is not known whether Hertz collected the Berlin prize money since the due date was over! He also made experiments to determine the velocity of electromagnetic wave propagation using a wire over a ground plane. With the transmission line open-ended at one end, he moved a detector along the lines and observed the distance between nulls is one half-wavelength and had a value of 2.8 m. [19]. Hertz calculated the frequency of the source form the estimate of the capacitance and the inductance of the dipole resonator. For the velocity, the product of *wavelength* × *frequency*, he arrived at a value of 2×10^8 m/sec not knowing of a computational error of $1/\sqrt{2}$ for the frequency. Without this error his result would have been 2.8×10^8 m/sec. After he published his results this error was pointed out to him in a letter from H. Poincaré in 1890 [19].

However, what Hertz missed was the core idea of the discontinuity in the displacement [13, p.193]. The quandary of Hertz forced him into an uneasy compromise with the traditional Helmholtzian view on charge. Hertz distinguished the free electricity from which one calculates forces, and which is alterable by non-conducting means, from the true electricity, which is alterable only by conduction. So, though Hertz referred the measure of true charge to the divergence of the displacement, he preserved Helmholtzian wording because he had not seen how to avoid it. Whereas, a Maxwellian would write apparent charge as $\nabla\bullet \boldsymbol{E}$, Hertz wrote of free charge and felt it necessary to retain the idea of bound charge to grant free charge physical significance, though he refused to consider why such a thing as bound charge exists. We see now the significance of Hertz's famed rejection of the Maxwellian distinction between electric intensity and displacement in the free aether. Without this distinction, it is impossible, in Maxwellian theory, to understand the existence of a charged surface in vacuum because charge is due to the discontinuity in the displacement. The fact that in free aether $\boldsymbol{D}$ reduces to $\boldsymbol{E}$ is merely a mathematical artifact. This is due to the definition of capacity of the aether being unity. The conceptual and physical distinction between displacement and intensity is still essential. Not knowing or understanding this distinction goes to the heart of Maxwell's theory. Hertz felt free to ignore it where it seemed mathematically to make no difference. In addition, Hertz started from the sources of the fields which were charges in electrostatics and currents in magnetostatics and not treat the potentials as fundamental quantities in analogy to a mechanical model as Maxwell did. Hertz showed that at a dielectric boundary the tangential components of the electric fields are continuous and the normal component is discontinuous. He obtained similar boundary conditions for a magnetic media.

In short [13], Hertz defined the electric and magnetic constants as unity in free space, thus eliminating displacement as a primary quantity. While this somewhat simplified the mathematical structure of Maxwell's theory, it was fatal to its dynamical basis. Heaviside raised this point with Hertz in 1890 after reading his first paper *On the Fundamental Equations of Electrodynamics*. He asked, *Can you conceive of a medium for electromagnetic disturbances which has not at least two physical constants, analogous to density and elasticity? If not, is it not well to explicitly symbolize them, leaving to the future their true interpretation?* Heaviside was calling on Hertz not to sacrifice for the sake of a small and perhaps illusory mathematical simplification, the dynamical aether, filled with stresses, strains, and stored energy on which Maxwell had built his theory. The British Maxwellians took this dynamical aether much more seriously than did their Continental counterparts, insisting that even on those points where Maxwell's theory required clarification and correction, this could and should be done without reducing the theory to a mere set of equations. The important modifications Heaviside had made to Maxwell's theory, including his abandonment of the potentials in favor of his four equations in the vector form that we call Maxwell's equations today, he said, meant as sustentative changes or new departures, but were directed solely at bringing out the leading points of the theory more clearly than had Maxwell himself. Both Heaviside and FitzGerald drew a careful distinction between *Maxwell's Treatise* and *Maxwell's theory* and said that Maxwell's book gave only an imperfect account of the real nature of the theory [13].

In 1886, Hertz observed while examining some of the apparatus used in lecture demonstrations that the oscillatory discharge of a Leyden jar or induction coil through a wire loop caused sparks to jump a gap in a similar loop a short distance away. He recognized this as a resonance phenomenon and saw that such sparking loops could serve as very sensitive detectors of oscillating currents and, thus, of electromagnetic waves. This provided him with the proper experimental tool which had eluded Lodge, FitzGerald, and others. His two papers: *On the finite velocity of propagation of electromagnetic actions* and *The forces of electric oscillations, treated according to Maxwell's theory* both published in 1888 vindicated Maxwell's theory and from then on it was accepted by the electromagnetic community.

Heaviside, on the other hand, differentiated between the absolute and the relative permeability and permittivity, defining the relative quantities as the ratio of the absolute value for a medium and that for free space. Heaviside rewrote Maxwell's equations in the vector form that we use today using the above modifications. There are conductors and non-conductors or insulators and since the finite speed of propagation in the non-conducting space outside conductors was unknown, attention was almost entirely concentrated upon the conductors and an assumed field which was supposed to reside upon or in them, and to move about, upon, or through them. And the influence on distant conductors was attributed to instantaneous action-at-a-distance, ignoring an intermediate agent. Maxwell explained these actions of the intermediate agent of an intervening medium being transmitted at finite speed. In Heaviside's words [18]:

What is Maxwell's Theory or what should we agree to understand by Maxwell's theory? The first approximation to the answer is to say there is Maxwell's book as he wrote it; there is his text and there are his equations, together they make his theory. But when we come to examine it closely, we find that this answer is unsatisfactory. To begin with, it is sufficient to refer to papers by physicists, written say during the twelve years following the first publication of Maxwell's Treatise, to see that there may be much difference of opinion as what his theory is. It may be, and *has been, differently interpreted by different men, which is a sign that it is not set forth in a perfectly clear and unmistakable form. There are many obscurities and inconsistencies. Speaking for myself, it was only by changing its form of presentation that I was able to see it clearly, and so as to avoid inconsistencies. It is therefore impossible to adhere strictly to Maxwell's theory as he gave it to the world, if only on account of its inconvenient form… But it is clearly not admissible to make arbitrary changes in it and still call it his. He might have repudiated them utterly. But if we have good reason to believe that the theory as stated in his treatise does require modification to make it self-consistent and to believe that he would have admitted the necessity of the change when pointed out to him, then I think the resulting modified theory may well be called Maxwell's.*

Maxwell defined the Ampère's law defining the electric current in terms of magnetic force. This makes the electric current always circuital implying that electric current should exist in perfect non-conductors or insulators. It is the cardinal feature of Maxwell's system. But Maxwell's innovation was really the most practical improvement in electrical theory conceivable. The electric current in a non-conductor was the very thing wanted to coordinate electrostatics with electrokinetics and consistently harmonize the equations of electromagnetic. It is the cardinal feature of Maxwell's system when dealing with Faraday's law where the voltage induced in a conducting circuit was conditioned by the variation of the number of lines of force through it. Instead of putting this straight into symbols, they went in a more round about way and expressed an equation of electromotive force containing a function called the vector potential of the current and another potential, the electrostatic, working together but not altogether in the most harmoniously intelligible manner – in plain English muddling one another. It is I believe a fact, which has been recognized that not even Maxwell himself quite understood how they operated in his general equations of propagation. We need not wonder, then, that Maxwell's followers have not found it a very easy task to understand what his theory really meant, and how to work it out. It was Maxwell's own fault that his views obtained such slow acceptance; and in now repeating the remark, do not abate one of my appreciations of his work, which increases daily.

To understand the problem, one must introduce the Maxwellian concept of a current. It was apparently not possible to incorporate conductivity into the dynamical structure of the Maxwellian theory [13]. Maxwell did not himself incorporate conductivity directly into the dynamical structure of the theory. Rather, he treated Ohm's law as an empirical, independent fact, and subtracted the electromagnetic intensity it requires from the induced intensity.

It is important to point out that Heaviside's duplex equations, as he called them, can be found in Maxwell's own writing in 1868, *Note on the Electromagnetic Theory of Light*. Maxwell gave a clear statement of the second circuital law relating the magnetic current to the curl of the electric force, crediting it to Faraday and asserting that it afforded "the simplest and most comprehensive" expression of the facts. But despite the advantages of this simple equation, particularly in treating electromagnetic waves, Maxwell did not use it at all in his *Treatise* in 1873 and it remained unknown even to his close students till 1890 when Maxwell's scientific papers [5, 6] were published. It is rather amusing to note that even Lodge (one of the Maxwellians) actually credited this equation to Heaviside in his Presidential Address to the Physical Society in February 1899, when the Irish physicist and mathematician Sir Joseph Larmor (1857–1942) pointed out to him that it was in the writing of Maxwell himself [9-11]!

In summary, why Maxwell's theory was not accepted for a long time by contemporary physicists was that there were some fundamental problems with his theory. Also, it is difficult to explain using modern terminology as to what those problems were. As stated in [13] *Maxwellian theory cannot be translated into familiar to the modern understanding because the very act of translation necessarily deprives it of its deepest significance, and it was this significance which guided British research. It assumed that field and matter can always be treated as a single dynamical system, subject to modification according to the circumstances.* In modern days, we reject this very basis of the Maxwellian theory! Maxwell's paper in 1862 *On Physical Lines of Force* described an elaborate mechanism for aether. He was deeply attached to the mechanism despite certain problems with it and remained throughout his life till 1879 strongly committed to the principle to model building. Yet, in 1864, his paper on *A Dynamical Theory of the Electromagnetics*, avoided specifying the structure of the aether, but nevertheless, presumed the field to be governed by what he called dynamical laws. This was done by treating ***H*** as a velocity and ***D*** as the curl of the corresponding mechanical displacement. Through such assumption, it does not mean that the true structure of the aether is fully understood. Maxwell's theory was based on the assumption that all electromagnetic phenomenon, including boundary conditions, can be obtained by applying Hamilton's principle to suitably chosen field energy densities which contain the appropriate electrical parameters like permittivity and permeability for the material medium. Modern theory implies that this can at best work on certain occasions as the macroscopic fields (***D***, ***H*)** are not a simple dynamical system but a construct obtained by averaging over the true state and the combined field vectors (***E***, ***B***) with material vectors (***P***, ***M***). Hence, where modern theory introduces the electron, the Maxwellian theory invented new forms of energy. This was possible because the Maxwellians were quite willing to invent modifications to the basic equations governing the electromagnetic field – as long as the results held up experimentally. Modern theory seeks unified explanations in an unmodifiable set of field equations coupled through the electron motions to intricate microphysical models. Maxwellian theory sought unity through a set of field equations coupled to Hamilton's principle. In Maxwell's theory, the goal was to create a theory of electromagnetism which made no use whatsoever of the microstructure of matter. Hence, the basic problem in understanding Maxwell's theory by a modern reader is to decipher what exactly did he mean by the words *charge* and *current*! Some of this confusion depends on Maxwell's having altered at least once his choice for the sign for the charge density in the equation which links it to the divergence of the electric flux ***D***. The conduction current in Maxwell's theory cannot be simply explained! The details of these subtleties may be found in [13]. Finally, as pointed out by Bruce J. Hunt, [9-16]: *When the 1880s began, Maxwell's theory was virtually a trackless jungle. By the second half of the decade, guided by the principle of energy flow, Poynting, FitzGerald and above all Heaviside have succeeded in taming and pruning that jungle and in rendering it almost civilized.*

In modern electrodynamics, we do not regard the field itself as a material structure, so we do not consider the stresses that may act upon it. Electromagnetic radiation, for example, transports energy and momentum but stresses arise only when the radiation impinges on material structures. The Maxwellians did not think in this way. For them energy inhomogeneity, whether matter is present or not, implies stress. Indeed after the discovery of Poynting's theorem, they realized that the free aether must be stressed when transmitting radiation, and so must move. In Maxwellian theory, the electromagnetic field transmits stress and is itself acted upon by stress. In modern theory, the field only acts; it is not acted upon [13]. There is an alternate point of view to this, however [20]. For example, one side of Poynting's theorem has [***J* • *E*]** while the other side has only fields. So one can say that the fields act on ***J*** or that the fields act on themselves, especially in cases like perfect electric conductors where no total work is done on the current and yet energy ends up in the fields.

In summary, the great difficulty with Maxwellian theory is that one cannot setup correspondences between mechanical and field variables which lead to consistent results unless one ignores conductivity, as illustrated by Heaviside. Heaviside's demonstration of this fact was based on the Green's potential function. He suggested using ***E*** as the velocity instead of ***H*** in the mechanical model and this requires a complete reconstruction of Maxwellian theory, as Maxwell did it the other way around [13].

The mathematical formulation of *macroscopic* electromagnetic phenomena was accomplished by Maxwell [21,22]. According to Maxwell's theory, electromagnetic radiation is produced by the oscillation of electric charges, the charges that produce light were unknown. Since it was generally believed that an electric current was made up of charged particles, Hendrik Altoon Lorentz in 1892 theorized that the atoms of matter might also consist of charged particles and suggested that the oscillations of these charged particles (electrons) inside the atom were the source of light. It should be noted that this occured prior to the experimental clarification of the basic properties of the electron; those were not measured until 1897 and 1898 when Joseph John Thomson succeeded in a determination of the charge to mass ratio and in a first measurement of the elementary charge *e*. Lorentz thus constructed a *microscopic* theory by using Maxwell's equations and adding to it an expression for the force which a charged particle experiences in the presence of electric and magnetic fields. This microscopic theory is a description of matter in terms of its charged atomic fragments, ions and electrons. The success of this microscopic theory lay in the proof first provided by Lorentz that the macroscopic Maxwell theory can be deduced from this microscopic theory by a suitable averaging process over the motion of the individual ions and electrons. The Maxwellian and Continental ideas were so profound that only Lorentz was able to retain the substantial character of charge while incorporating certain Maxwellian elements that did not violate basic preconceptions. Lorentz's theory computes interparticle actions directly by using retarded forces and it employs careful microphysical averaging. It is interesting to note that the Lorentz force equation is very similar to Maxwell's equation (D) as described in section 3. In 1896 Pieter Zeeman, then an experimental physicist in Leiden, made a surprising discovery: the splitting of spectral lines by a magnetic field (This in fact is similar to the Stark effect founded by Johannes Stark in 1913 and independently by Antonio Lo Surdo, the same year, namely the splitting of the spectral lines due to a static electric field). Lorentz was able to explain the new phenomenon with his electron theory. He concluded that the radiation from atoms consisted of negatively charged particles with a very small mass. In 1902 Zeeman and Lorentz jointly received the Nobel Prize in Physics.

The Maxwellian and Continental ideas were so profound that only Lorentz was able to retain the substantial character of charge while incorporating certain Maxwellian elements that did not violate basic preconceptions. Lorentz's theory computes interparticle actions directly by using retarded forces and it employs careful microphysical averaging. However, Lorentz's theory very much obscures the difference between ionic motions and the basic field equations [13].

From 1894 to 1897, the British electromagnetic theory abandoned the basic principles of Maxwellian theory and the entire subject was reconstructed on a new foundation – the electron, developed by Joseph Larmor in consultation with George FitzGerald [13]. Larmor proposed that the aether could be represented as a homogeneous fluid medium which was perfectly incompressible and elastic. Larmor believed that aether was separate from matter and that matter consisted of particles moving in the aether. Larmor believed the source of electric charge was a "*particle*" (The concept of a quantum electrical charge had been theorized on several occasions beginning in 1838, including by Irish physicist George Johnstone Stoney in 1874, who introduced the name *electron* in 1894). Parallel to the development of Lorentz ether theory, Larmor published the complete Lorentz transformations in the *Philosophical Transactions of the Royal Society* in 1897 some two years before Hendrik Lorentz (1899, 1904) and eight years before Albert Einstein (1905) [21, 22]. Larmor predicted the phenomenon of time dilation, at least for orbiting electrons, and verified that the FitzGerald-Lorentz contraction (length contraction) should occur for bodies whose atoms were held together by electromagnetic forces. Larmor thus effected a revolution in the Maxwellian theory, one in which the electron had become the fundamental for generation of the field. Heaviside felt that Larmor's idea of the electron was not sufficiently fundamental. The major impact of Larmor's theory was the destruction of the idea that continuum theory can serve as a sufficient basis for electromagnetism. Maxwell's theory with its fundamental assumption that the electromagnetic field can be subjected to precisely the same type of analysis as the material continuum was an artifact after 1898. This was the second conceptual modification of Maxwell's theory after Hertz, and Heaviside even though the basic equations still remained the same. Larmor's student, John Gaston Leathem, connected mathematically displacement with polarization by writing $\boldsymbol{D} = \boldsymbol{E} + \boldsymbol{P}$ and thus effectively destroying the Maxwell-FitzGerald theory of using Hamilton's principle as a starting point based on a continuous energy function [13].

From this time onwards, future work on electromagnetism, in Britain and in the Continent, depended directly on microphysics based on the electron rather on the macrophysics. Hence there was a complete divorce between the matter and the field [13]. Thus, the nature of conduction, the stumbling block in the Maxwell's theory, led to the final modification by Larmor and Lorentz by incorporating the microscopic view of matter through the introduction of the electron as the source of charge, the flow of which results in a current. These sources now produce the fields.

In short, the current perception of Maxwell's theory consists of Maxwell's system of equations supplemented by his radical concept of displacement current. A relevant question here: Is this displacement current has a physical reality? If so, then how is it that it produces radiation similar to that by conduction current? Currently accepted form of Maxwell's equations as obtained through the work of Hertz, Heaviside, Lodge, Fitzgerald, Lorentz and Larmor are given as follows by these equations which are available in any modern text book on electromagnetic theory: $\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H}$; $-\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \boldsymbol{E}$; $\nabla \bullet \boldsymbol{D} = q_v$; $\nabla \bullet \boldsymbol{B} = 0$; $\nabla \bullet \boldsymbol{J} = \frac{-\partial q_v}{\partial t}$; along with $\boldsymbol{B} = \mu \boldsymbol{H}$ and $\boldsymbol{D} = \varepsilon \boldsymbol{E}$. Equation of continuity here is included for completeness.

5. Conclusion: In this article, we first have started with a brief life story of James Clerk Maxwell followed by his significant achievements, with emphasis on his work in electromagnetics. In particular, we discussed in details how his original equations and concepts were modified by Hertz, Heaviside, Lodge, Fitzgerald, Lorentz and Larmor, towards the evolution of what we accept know as Maxwell's equations and Maxwell's electromagnetic theory.

The undeniable influence of Maxwell's equations and other related ideas to the discipline of electrical engineering, and the influence of his other significant contributions in science make him one of the great scientists of all times.

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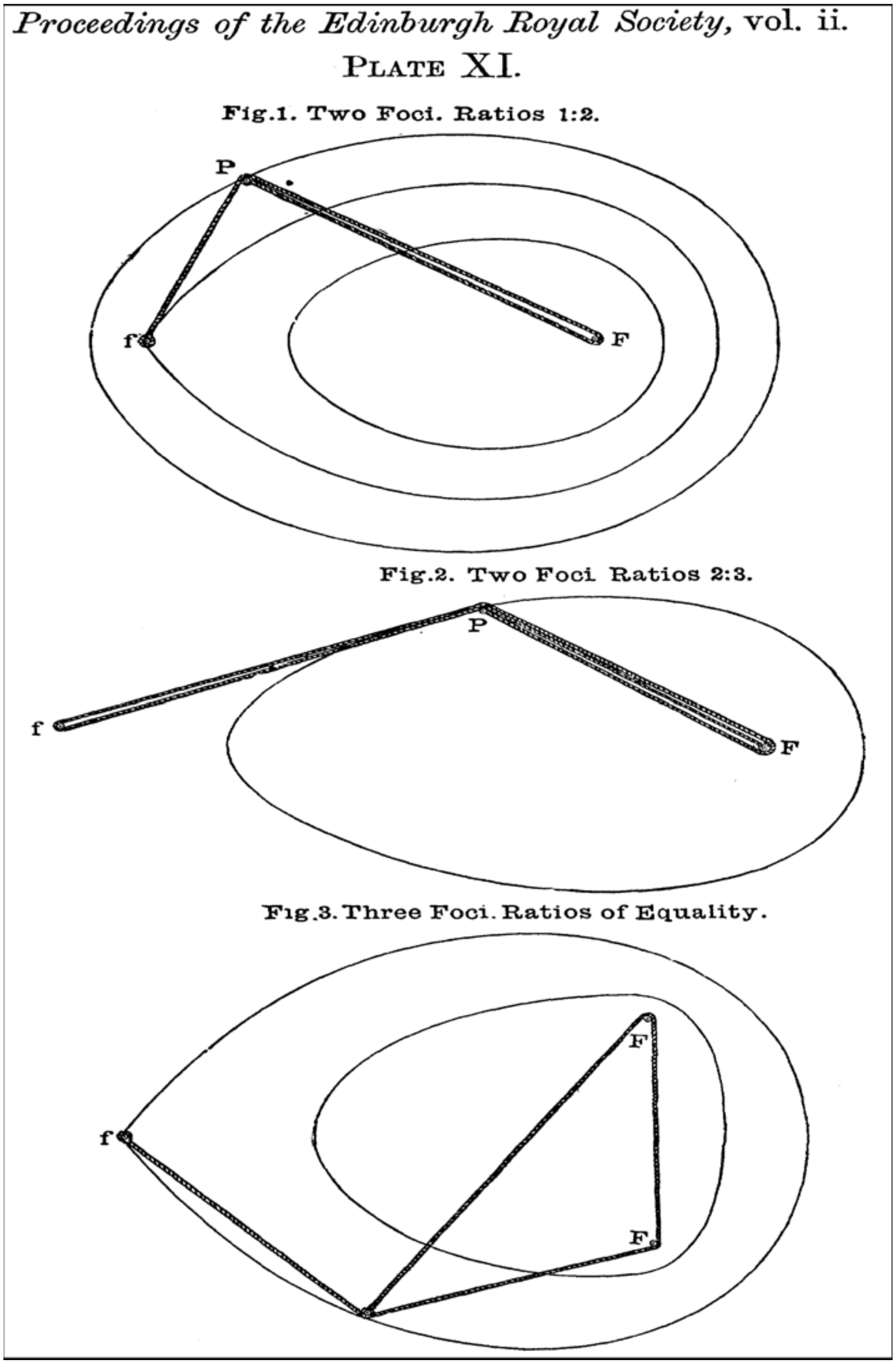

Figure 1. Construction of curves with multiple foci using pins, pencil and a thread.

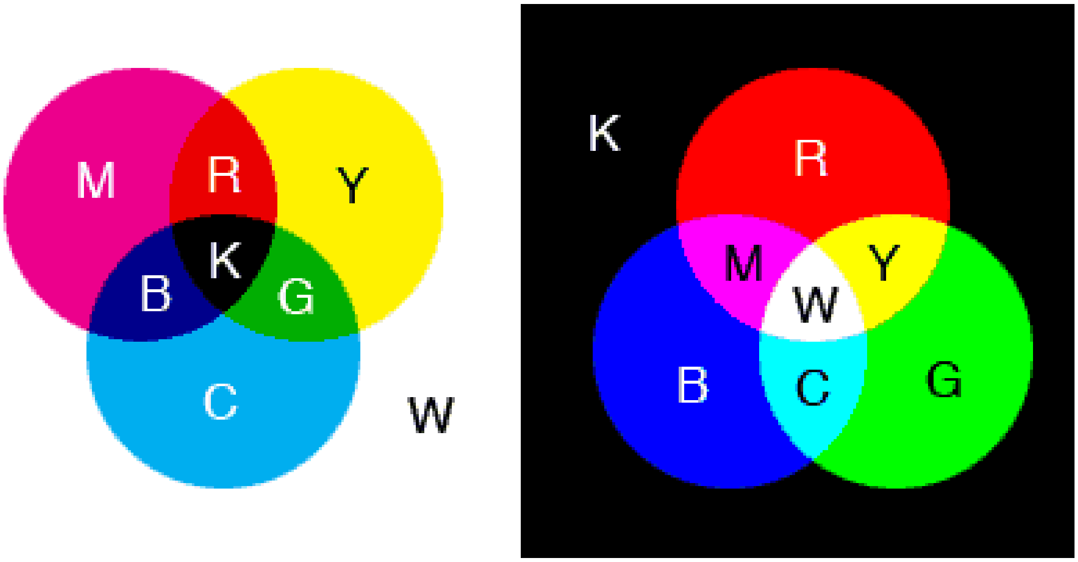

Figure 2: Generation of any color using the painters subtractive process(left) and using different colored light for the additive process in generating any color of the spectrum(left). The three primary colors for each case is however different (ian-albert.com/graphics/addsub**color**.php).

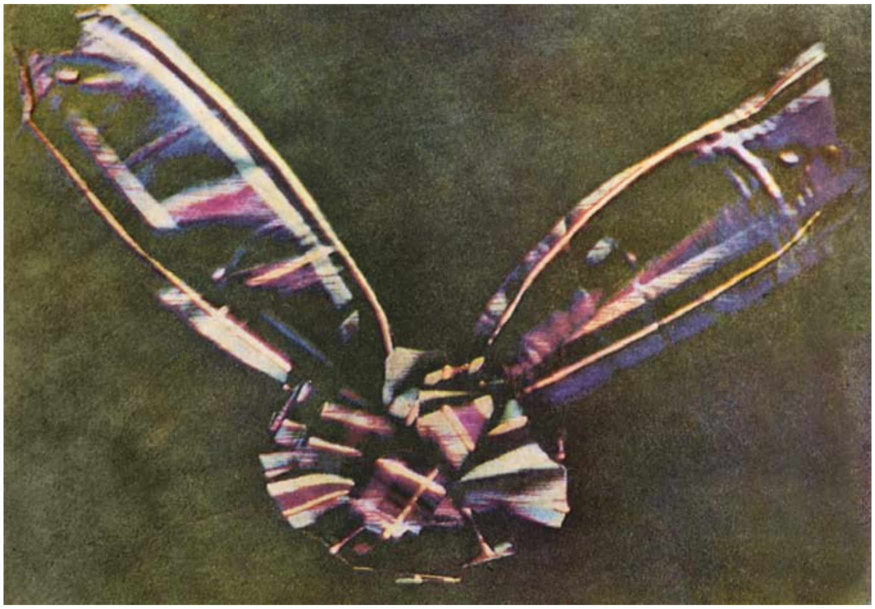

Figure 3. The first color photograph of a Tartan ribbon taken by Thomas Sutton for Maxwell in 1861.

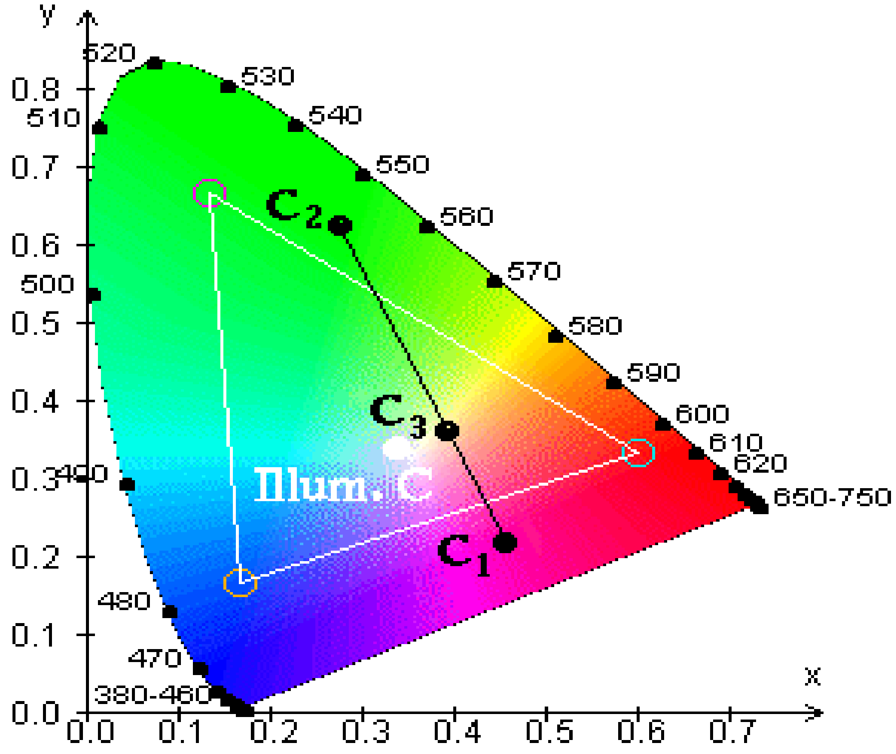

Figure 4. The color triangle or a chromatist triangle (http://en.wikipedia.org/wiki/Color).

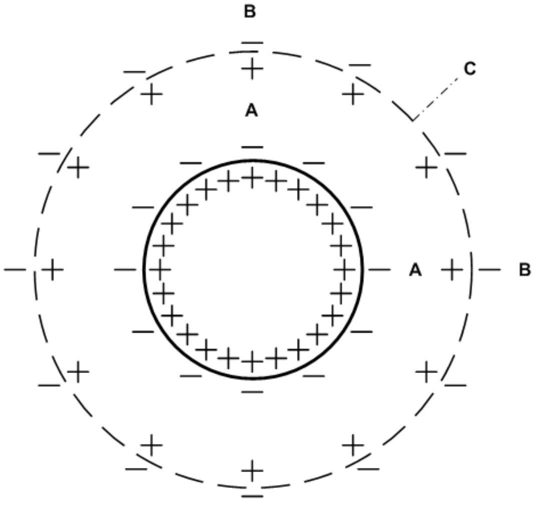

Figure 5a. Modern picture for a conducting sphere embedded in a dielectric.

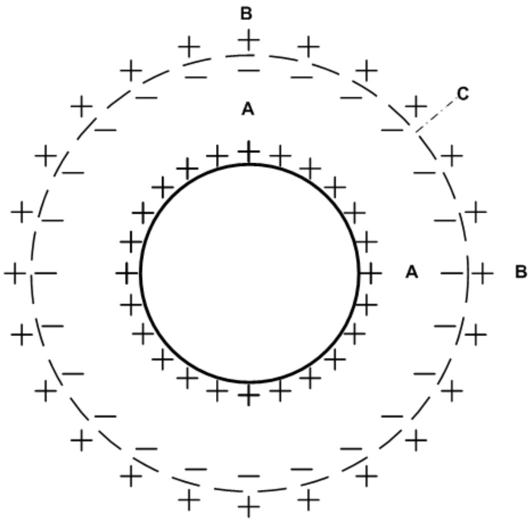

Figure 5b. Maxwellian picture for a conducting sphere embedded in a dielectric.

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She has authored a total of 490 scientific publications: 6 books and 23 contributions (chapters or articles) for books published by international editorial companies, 13 contributions for academic books and notes, 62 papers in scientific journals, 234 papers in international conferences, symposiums, and workshops, 71 papers in national conferences and 81 project reports, short course notes, and other publications. She has coauthored 2 European and USA patents which have been extended to other countries and several software packages for the analysis and design of microwave and millimeter wave passive components, antennas and antenna arrays, as well as computer aided design (CAD) of advanced filters and multiplexers for space applications which are been used by multinational companies. She has delivered numerous invited presentations and seminars. She has lectured in more than 50 short courses, some of them in the frame of Programs of the European Community and others in conjunction with IEEE (The Institute of Electrical and Electronic Engineers) International AP-S (Antennas and Propagation Society) Symposium and IEEE MTT-S (Microwave Theory and Techniques Society) Symposium and others IEEE Symposia. She has participated at different levels (principal investigator or researcher) in a total of 81 research projects and contracts, financed by international, European, and national institutions and companies, among them: the National Science Foundation, USA; the European Office of Aerospace Research and Development of the Air Force Office of Scientific Research (one of the Air Force Research Laboratory Directorates), USA; the European Union; Spain Inter-ministry Commission of Science and Technology (CICYT), Spain Ministry of Education and Culture, Spain Ministry of Science and Innovation, and Council of Education of the Regional Government of Madrid. She is a registered engineer in Spain. She has received two individual research awards.

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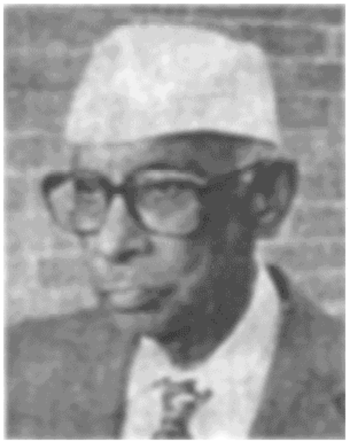
Since 1989, she has served IEEE under different volunteer positions: Vice Chairperson and Chairperson of IEEE Spain Section AP-S/MTT-S Joint Chapter, President of IEEE Spain Section, Membership Development Officer of IEEE Spain Section, member of IEEE Region 8 Committee, member of IEEE Region 8 Nominations and Appointments Subcommittee, Chairperson of IEEE Region 8 Conference Coordination Subcommittee, member of IEEE Women in Engineering (WIE) Committee, liaison between IEEE WIE Committee and IEEE Regional Activities Board, Chairperson of IEEE WIE Committee, member of IEEE Ethics and Member Conduct Committee, member of IEEE History Committee, member of IEEE MGAB (Member and Geographic Activities Board) Geographic Unit Operations Support Committee, and member of IEEE AP-S Administrative Committee. Presently she is serving as member of IEEE Spain Section Executive Committee (officer for Professional Development), member of IEEE MTT-S Subcommittee # 15, and member of IEEE AP-S Transnational Committee. In December 2009 she was elected 2011 President of IEEE AP-S Society, acting as Vice President during 2010.



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